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DISCUSSION

Introducing Mechalus to Geometry. In ancient times the few greatest scholars of a generation studied geometry; it was a senior study at Yale until 1742; but now we expect Mechalus Pezanoski, aged 16, who came over in the steerage from Poland when he was a baby, to grasp demonstrative geometry in one year. Obviously, our task of teaching that geometry is vastly different from that of our predecessors.

Mechalus was sent to school because the state law requires his presence there until he is 17. He has no desire to study geometry in order that he may learn formal reasoning; he lives in a concrete world dominated by the movies and his transition to the rarified atmosphere of abstract thought must be gradual or his brain will become temporarily paralyzed. Happily compasses, a protractor, and ruler are concrete things and if they are used to construct the same geometric figures he met in elementary mathematics, his introduction to the new subject is safely begun.

The next difficulty he will encounter is the group of necessary preliminary axioms and definitions. Several axioms he has already met in algebra; properly encouraged he may formulate others from them, and vivid illustrations will fix the most difficult in his mind. For instance, if John, the tall athlete, medium-sized Harry and Fred, the class pigmy, are lined up in a row and renamed "things," Mechalus will see very clearly that if the first of three things is greater than the second, and the second is greater than the third, the first is greater than the third.

When after many intuitional exercises the formal demonstration of a theorem actually begins, Mechalus is likely to look at his teacher with a feeling akin to pity when she proves with much labor that two certain triangles are congruent when he knew they were congruent by merely looking at them. But he may be induced to feel there is some need for proof if several eye-deceiving diagrams are drawn which appear to have one relation and may be easily proved to have another. He then may be quite willing to cut out paper triangles whose specified measurements show the parts which are given equal, and use them with his newly acquired rules for the new game to prove that these triangles may be made to fit and are therefore congruent.

He next encounters his old enemy English when he finds that he must write out this proof. But here again his teacher proves to be an ally and gives him the following skeleton outline which he uses for every proof:

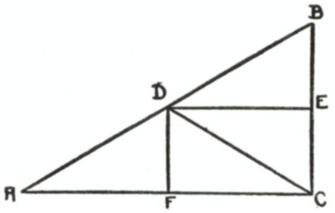
1. Statement of theorem to be proved.
2. Figure.
3. Given.
4. To prove.
5. Proof:

   Arguments   Reasons.
6. Statement of theorem proved.

Together they work out the complete written proof and Mechalus copies it in his note book for a model.

Much the same method of procedure is followed for several days; gradually Mechalus finds it unnecessary to cut out paper triangles, he can see them plainly enough in the drawings he makes; gradually the teacher lessens the amount of proof in the models put on the blackboard and Mechalus completes them in his notebook. In order that this notebook may be a trustworthy guide he hands it in each week for correction.

Mechalus enjoys the many easy exercises which are given beginning with the first theorem, and later in his course feels like a creator of mathematics if he can discover for himself theorems which are found on later pages of the textbook. To this end the teacher may make an occasional assignment like this:



Given the right triangle *ABC* with *CD* connecting *D* the midpoint of *AB* with *C, DE* parallel to *AC,* and *DF* parallel to *BC.*

State all the theorems that appear to be true and try to prove them.

If geometry continues to compete successfully with the movies the recitations must show variety. Historical bits may be used to throw glamour on cut and dried theorems and Mechalus is entranced with the report of the Bright Boy who tells about Pythagoras and his mystic fraternity. On a Monday morning he may be awakened by a few minutes spent on foolish thought problems like the following: If 2 horses can jump a ditch 3 feet wide, how wide a ditch can 8 horses jump? Another good alarm clock is the apparently correct proof of the obviously impossible facts that every triangle is isosceles and that an obtuse angle equals a right angle.

Algebra is the first aid for many theorems. For example, in the problem, "To construct a square equal in area to a given triangle," if Mechalus makes use of his old friend x for the unknown side of the square, a and b for the altitude and base of the triangle, he can with little difficulty translate his problem into the equation x^2 equals $\frac{1}{2}ab$. This easily suggests the proportion $\frac{1}{2}a/x$ equals x/b, which, translated into geometry states that the side of the square may be found by constructing the mean proportional between $\frac{1}{2}a$ and b.

But Mechalus is as lazy as he dares to be; he is quite willing to take a chance that the Bright Boy will be called upon to recite the more difficult theorems. In order to make him feel responsible for each of the fundamental theorems the following scheme was devised and worked quite successfully:

At the beginning of each small unit of work the most important theorems were assigned to be recited within a specified time by each member of the class. After the pupil had mastered the theorem from the meagre suggestions in the text, he put the figure on the board and wrote his name on a slip of paper on the teacher's desk to show he was ready to recite. The recitation was given to the teacher or to one of her assistants, the Bright Boys, who had already given a perfect demonstration of the proposition.

The advantages of the plan were several. Every pupil knew that he must recite every assigned theorem, and therefore attempted to prepare it; every pupil had the ambition to be an assistant teacher so he aimed for a perfect instead of a passing demonstration; every pupil could progress at his own pace, and ample provision could be made for additional work for the Bright Boy; make-up work for absences could be more easily checked up; initiative was developed and enough interest was aroused so that the closing bell was greeted with groans instead of cheers.

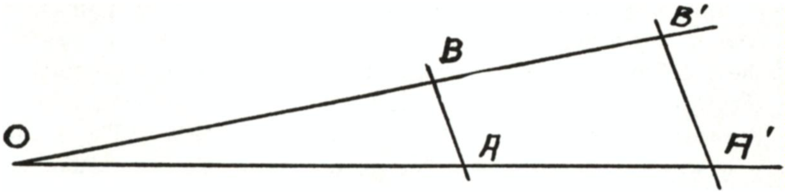
The last half or two-thirds of the class time was usually devoted to this sort of free-for-all recitation; the first part was spent in the conventional manner of anticipating difficulties, correcting recurring errors, reciting and discussing originals and theorems which comprised the additional assignment for that day's work.

By this time Mechalus has become sufficiently interested in geometry and accustomed to its abstract ideas and formal reasoning so that he bears the necessary if unpleasant reviews, outlines, drills, and even examinations with fortitude if not with pleasure. MARY A. POTTER.

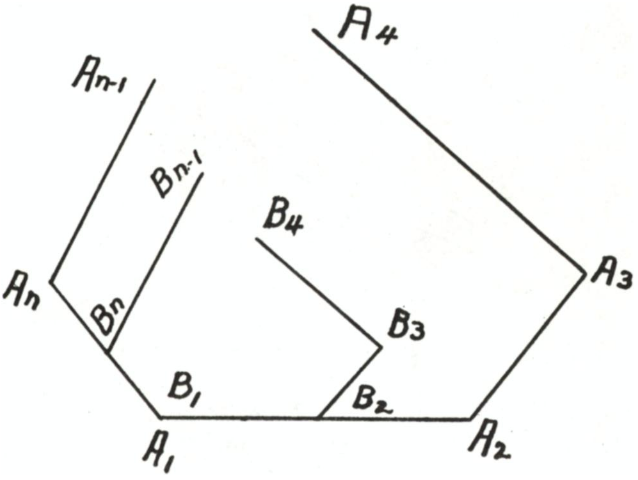
Racine, Wisconsin.

Notes on Mr. Evans' Paper in the March Teacher. One cannot help but feel grateful to Mr. Evans and the TEACHER for a paper which does not hesitate to break with tradition. We are the more grateful because of the recent command to fall down and worship tradition—or international authority, as it is now called.

In a paper having such a large sweep it is to be expected that there will be matters which will not appear to all to hang well



together. One such discrepancy is Mr. Evans' point of view concerning what he calls "perspective" position and the early stages of his plan. He says, "No change in the definition of similar triangles; leave that as it is." *But is it not a fact that in order to prove the theorems on similar triangles this same* *perspective position must be employed.* And if the bond *must* be formed *in the beginning* between the definition of similar triangles and the perspective position is it not better psychology to continue this bond than to break it. The writer maintains that the figure below is essentially in perspective position.



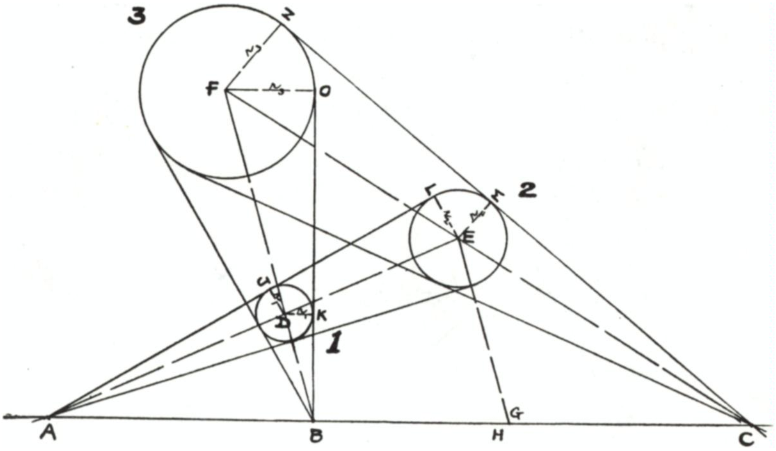
Furthermore, in two dimensions it may be doubted whether or not there is any real need ever to break the bond between similar polygons and perspective position. Mr. Evans condemns this bond as unnatural and inadequate. His words are, "Against the general use of such a definition and method, however, is the fact that similarity belongs to the figures themselves, and not to their relative position." As to *method,* and with the *present definition,* it seems that Mr. Evans' position may be questioned on the basis of fact. As a method the perspective position seems entirely adequate, as in the last figure.

Montclair, N. J. H. F. HART.

Original Solution in Plane Geometry.

Problem: The points of intersection of external tangents drawn between any two of three circles of different sizes, in turn, lie on a straight line passing through these points.

Given the three circles No. 1, No. 2 and No. 3; and the external tangents between the circles No. 1 and No. 2, intersecting at the point A; those between circles No. 1 and No. 3 intersecting at the point B; those between the circles No. 2 and No. 3 intersecting at C.



To Prove: That the points A, B and C lie on a straight line ABC passing through the points A, B and C.

Proof: Let r_1, r_2 and r_3 be the radii of the circles No. 1, No. 2 and No. 3, respectively, and D, E and F their centers.

In the similar triangles ADJ and AEL,

$$(1) \quad \frac{AD}{AE} = \frac{r_1}{r_2}$$

In the similar triangles BDK and BFO

$$(2) \quad \frac{BD}{BF} = \frac{r_1}{r_3}$$

In the similar triangles CEM and CFN

$$(3) \quad \frac{CE}{CF} = \frac{r_2}{r_3}$$

Combining equations (1) and (3) by multiplication, we have

$$(4) \quad \frac{AD}{AE} \times \frac{CE}{CF} = \frac{r_1}{r_2} \times \frac{r_2}{r_3} = \frac{r_1}{r_3}$$

Equating equations (2) and (4), we have

$$\frac{BD}{BF} = \frac{AD}{AE} \times \frac{CE}{CF} \quad \text{or}$$

(5) $$\frac{DB \times AE}{AD} = \frac{CE \times FB}{CF}$$

From the point E, draw the line EG parallel to the line FB and meet the straight line AB produced at the point G.

Then in the similar triangles ADB and AEG

(6) $$\frac{AD}{AE} = \frac{DB}{EG}$$

Connect the points B and C by a straight line. Draw the line EH parallel to FB and to meet the straight line BC at the point H.

Then in the similar triangles CEH and CFB.

(7) $$\frac{CE}{CF} = \frac{EH}{FB}$$

From equation (6)

(8) $$EG = \frac{DB \times AE}{AD}$$

From equation (7)

$$EH = \frac{CE \times FB}{CF}, \text{ but from (5) } \frac{DB \times AE}{AD} \times \frac{CE \times FB}{CF}$$

Therefore, $EG = EH$.

But EG and EH are equal and parallel; therefore EG and EH coincide and the points G and H are one and the same point.

Hence, the straight line BG (BH) is common to both the straight line AB produced and the straight line BC.

Hence, the straight line BC is the continuation of the straight line AB.

Hence, the points A, B and C lie on the same straight line ABC. Q. E. D.

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An extract from the autobiography of Herbert Spenser, page 187, volume 1, adds interest to this proof: "It may be remembered that an early chapter states that when seventeen, I hit on a geometrical theorem of some interest. This remained with me in the form of an empirical truth; but during the latter part of my residence in Worcester, responding to a spur from my father, I made a demonstration of it, and now, that it had developed this form, it was published in the *Civil Engineer and Architects Journal* for July, 1840. It is produced in Appendix B. I did not know at the time that this theorem belongs to that division of mathematics at one time included under the name of Descriptive Geometry, but known in more recent days as the geometry of position—a division which includes many marvellous truths. Perhaps the most familiar of these is the truth that if to three unequal circles anywhere placed, three pairs of tangents be drawn, the points of intersection of the tangents fall in the same straight line—a truth which I never contemplate without being struck by its beauty at the same time that it excites feelings of wonder and of awe; the fact that apparently unrelated circles should in every case be held together by the plexus of relations, seeming so utterly incomprehensible. The property of a circle which is enumerated in my own theorem, has nothing like so marvellous an aspect but is nevertheless sufficiently remarkable.''

Professor Hedrick's Report on the Function Concept in Elementary Mathematics. Of the various preliminary reports which have been issued by the National Committee on Mathematical Requirements, none is more provocative of thought than the report on the function concept. This was to be expected by those who have the good fortune of personal acquaintance with the author. In direct ratio to his high standing among mathematicians in this country, and his familiarity with the various problems in the field of secondary instruction, is the degree of boldness exhibited by one who ventures to disagree with him at certain points.

Or rather, not so much to disagree with his conclusions as to question the practicability of many of his recommendations, however desirable they may be from the point of view of an inclusive study of the whole problem. In other words, it is the desire of the writer to ascertain just what Professor Hedrick is ''letting us in for.''

He takes as his text a quotation from the first report issued by the committee, as follows "*The one great idea* which is sufficient in scope to unify the course is that of the functional relation."

Let it be said at once that this is the one idea which at the present time in secondary mathematics is most likely to be thrown out of the window.

Is there any one idea about which the present course in elementary algebra (one or two years), is centered? I think that there is, and that it is the idea of *transformation.* Is it not true that the so-called algebraic *operations* are really transformations in accordance with laws which define properties? The whole emphasis in the past has been upon the universal character of these laws.

(Or rather, it should have been so. In too many instances the laws have been lost sight of in a welter of technique, with examples and methods based for the most part upon imitation.)

Elementary algebra has in the past been almost wholly static. Efforts to render it otherwise have been feeble and generally futile, for the reason that to do so successfully involves the upsetting of the whole program. Professor Hedrick shows clearly enough what is the elementary approach to the subject. The writer's point is that it is doubtful if occasional examples would serve any very useful purpose unless the whole subject were recast along the lines recommended.

If this were done, we would have a subject much more interesting than our present possession. Professor Hedrick indicates that it is not necessary to *wait* for the study of graphs before introducing the function idea. This is true, but in a course which has been unified about the functional relation the graph would be the *first* topic and others would cluster around it. Negative numbers, irrational numbers, and various algebraic forms would then be interpreted in their relation to the continuity of the graph. This development is, in the opinion of the writer, entirely within the compass of the secondary education from its beginning. But the usual sequence of topics would be sadly shaken. And unless the time limits were greatly extended, much of what is now regarded as essential to a secondary course, in the way of extrication of the ''unknown'' from an intricate mass of symbolism, must of necessity go by the board.

To illustrate—ratio and proportion and the progressions would be among the first topics considered in the ninth year course, with fractional and negative exponents an immediate sequence of the negative number. The polynomial, its factors, and operations upon it would appear much later and be stressed very lightly. Logarithms would come early in the course, and algebraic fractions near the end. Functions which in general are defined by algebraic fractions are rather difficult to handle.

The graph at present occupies a somewhat anomalous position in the secondary course. It is usually presented as the picture, not of a function but of an equation in two variables in which the functionality of both variables is implicit. For this reason the relation of the graph to the algebraic form appears to the student as more or less accidental.

If the function (and presumably its graph) is to be the one great idea which unifies the course, it should be developed much further in secondary mathematics than is done at present, or is suggested by Professor Hedrick. After all, the *mathematics* of the function idea is concerned with the definition of the function. To fail to do this, so to speak, takes all of the fun out of the function! It is all very well to say, for example, that horse-power is a function of ''R.P.M.,'' but the question is ''What function?'' and the topic loses interest unless the answer is forthcoming. In the case of the steam engine it is a linear function. In the case of a six-cylinder automobile engine it is something quite different, to say the least. Progress in the subject would be represented in a measure by the ability to define functions, and this would lead to such topics as recurring series, and the method of finite differences, presumably in the fourth high school year, and to the early consideration of Fourier series and other series in the college course. The writer would state in passing that he has taught the first two topics mentioned in a fourth year high school course to the great satisfaction of the students, and to his own, but without any alarming degree of logical precision!

And this brings us to the crux of the whole question. In the past it has been incumbent upon the secondary teacher (ostensibly) to set for his students a standard of logical precision. It was this thing and nothing else that gave Euclid such supreme importance in the eyes of the typical English schoolmaster. It is this which has led eminent lawyers, physicians, and clergymen to descant upon the glories of plane geometry in early schooling, in spite of the fact that in most American texts Euclid has been so terribly mishandled that the vaunted logical precision of elementary geometry is a philosophical shriek.

And the spirit of Euclid has dominated the teaching of algebra in America in spite of greater abuse in point of precision. Up to fifteen years ago, so the best authority informs us, the study of Euclid preceded that of algebra in English schools, for the most part. In America the order of the subject was reversed, but the aims and ideals were retained, again for the most part. The whole spirit of Felix Klein's proposed reforms in secondary mathematical teaching looked away from the hard-boiled formalism of an earlier generation, and toward a broadening of the vision of the significance of mathematics in the works of nature and in the field of human endeavor. That Klein himself recognized the dangers lurking in any procedure in this direction is evidenced by a conversation with him, reported by Professor D. E. Smith, touching upon this very point. The vital point remains, nevertheless. If the secondary course is to be centered about the function concept, considerations of logical precision will of necessity be in no small measure postponed to advanced courses in college and graduate school.

And this brings us to geometry, where the writer must take definite leave of Professor Hedrick. He apparently thinks that it is practicable to mention functions incidentally to the customary development of plane synthetic geometry. This would only make the usual geometric confusion worse confounded. We have dropped completely (in America) Euclid's theory of proportion and measurement, a theory which, as Dean Fine has pointed out, is perhaps his most brilliant achievement, foreshadowing the recent researches of Kronecker. But we have retained, out of respect for the classical tradition, the previous development necessitated by this theory, and have worried ourselves half sick over the axiom of parallels. If geometry is to share in the unification around the function concept, it means at the very outset the establishment of a correspondence between points and number forms, and that means the basing of the subject upon Cartesian analysis. An *elementary* analytic geometry is no more difficult than an *elementary* synthetic geometry. The whole question turns upon what is elementary, and that is a matter to be decided by experience. Geometry then assumes its proper place in the mathematical sequence. Probably the experiment has never been tried. But is it not likely that the general idea of a set of transformations is reducible to elementary terms? In this way the function concept would dominate the subject, and motion would assume its rightful place in the geometric scheme, instead of being, as it is now, a sheet anchor, under the alias of "superposition," to save from logical shipwreck, in spite of the fact that in a geometry which is wholly static, as ours sets out to be, motion has no place at all.

In such a development, Klein's definition of a geometry as a set of properties which remain invariant under a given transformation takes on an elementary significance. We would have a geometry of parallelism, of symmetry, of point reflection, of areas, of similarity, and possibly some others. The preparation of a text of this character would be an interesting experiment. Judging by past performances, however, it will be some time before such a text has any vogue in secondary circles. In the meantime, the function concept may as well be left out of a secondary course in geometry.

As far as trigonometry is concerned, the function concept lies at the basis of the subject, and there is no ancient canonical sequence to hinder its free development. If, in actual classroom practice, the trigonometric functions are treated merely as the properties of fixed angles, it is an evidence of imperfect acquaintance with the subject on the part of the instructor or the textbook writer. There are many evidences of this in certain instances. The report does well to remind teachers what a trigonometric function really is.

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